Lecture 2

Revison

$$\bar{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial t}\vec{k}$$

napla operater
del operater
delta operater

delta $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$\bar{\nabla}\phi$ grad ϕ (vector)

$\bar{\nabla}\cdot\bar{F}$ divergance div $\bar{F}$ (scalar)

$\bar{\nabla}\times\bar{F}$ curl curl $\bar{F}$ (vector)

$\bar{\nabla}\cdot\bar{V} = 0 \Rightarrow$ solenoidal field (أنبوبي)

$\bar{\nabla}\times\bar{F} = 0 \Rightarrow$ irrotational (غير دوراني)

$$\bar{\nabla}(\varphi_1 + \varphi_2) = \bar{\nabla}\varphi_1 + \bar{\nabla}\varphi_2$$

الانحدار للمجموع قياسيين

اثبات

$$\bar{\nabla}(\varphi_1+\varphi_2)=\bar{\nabla}\varphi_1+\bar{\nabla}\varphi_2$$

$$\left(\frac{\partial}{\partial x}\vec{i}+\frac{\partial}{\partial y}\vec{j}+\frac{\partial}{\partial z}\vec{k}\right)(\varphi_1+\varphi_2)$$

$$\bar{\nabla}\cdot(\bar{A}_1+\bar{A}_2)=\bar{\nabla}\cdot\vec{A}_1+\bar{\nabla}\cdot\vec{A}_2$$

$$\vec{\nabla}\times(\vec{A}_1+\vec{A}_2)=\vec{\nabla}\times\vec{A}_1+\vec{\nabla}\times\vec{A}_2$$

Ex.

$$\bar{\nabla}\cdot(\varphi\vec{A})=\varphi\,\bar{\nabla}\cdot\vec{A}+\vec{A}\cdot\bar{\nabla}\varphi$$

اشتقاق

Ex.

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A})=\begin{vmatrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ A_1 & A_2 & A_3\end{vmatrix}$$

$$\boxed{\therefore \vec{A}=A_1\vec{i}+A_2\vec{j}+A_3\vec{k}}$$

Ex.

$$\vec{\nabla}\times\vec{\nabla}\varphi=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ \frac{\partial\varphi}{\partial x} & \frac{\partial\varphi}{\partial y} & \frac{\partial\varphi}{\partial z}\end{vmatrix}=\vec{0}$$

Ex.

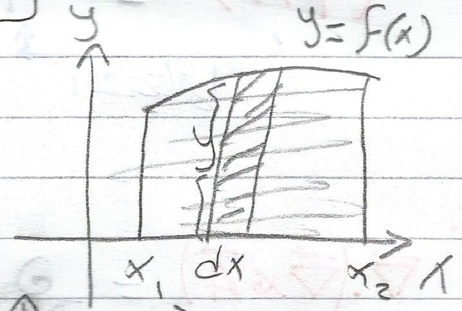
$$\vec{\nabla}\cdot\vec{\nabla}\varphi = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z}\vec{k}\right)\cdot\left(\frac{\partial\varphi}{\partial x}\vec{i} + \frac{\partial\varphi}{\partial y}\vec{j} + \frac{\partial\varphi}{\partial z}\vec{k}\right)$$

$$= \left(\frac{\partial^2\varphi}{\partial^2 x} + \frac{\partial^2\varphi}{\partial^2 y} + \frac{\partial^2\varphi}{\partial z^2}\right) = \Delta\varphi$$

Line integral التكامل الخطي

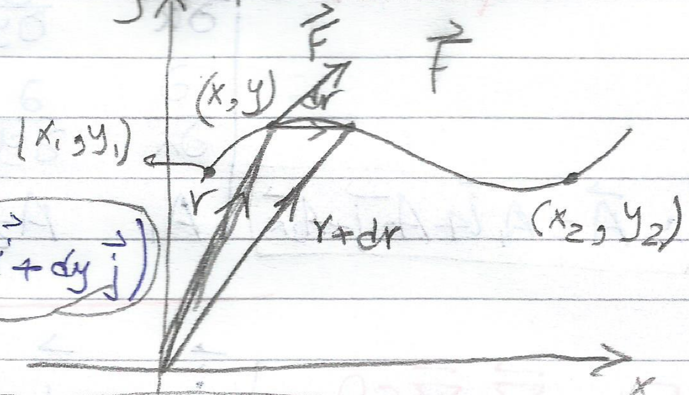
independant varibale X Riemaun integral
dependant varibale y

$$\int y(x)\,dx$$



$$\int_{(x_1,y_1)}^{(x_2,y_2)} \vec{F}\cdot d\vec{r} \equiv$$

$$= \int_{(x_1,y_1)}^{(x_2,y_2)} (F_1 i + F_2\vec{j})\cdot(dx\vec{i} + dy\vec{j})$$



$$\boxed{\vec{r} = x\,i + y\vec{j} \qquad d\vec{r} = dx\,i + dy\vec{j}}$$

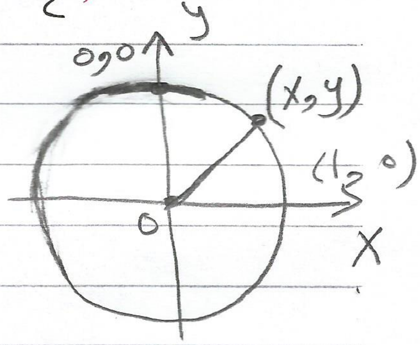
$F_1 = F_1(x, y$ $\qquad F_2 = F_2(x, y)$

$$\int_{(x_1, y_1)}^{(x_2, y_2)} F_1\,dx + F_2\,dy$$

Evaluate The Line integral $\left(\int 2xy\,dx + (x^2+y^2)\,dy\right)$ on the curve given by The equation

$(x = \cos t)$ & $(y = \sin t)$; $(0 \le t \le \frac{\pi}{2})$

Sol



$$\int 2xy\,dx + (x^2+y^2)\,dy$$

$$= \int 2xy\,dx + \int x^2+y^2\,dy$$

بالتحويل لكون المتغير X

$x^2 + y^2 = 1$

$y = \pm\sqrt{1-x^2}$

$$= 2\int_1^0 x\sqrt{1-x^2}\,dx + \int_0^1 dy$$

$\boxed{\text{أكمل في المنزل}}$

$x = \cos t \qquad dx = -\sin t\,dt$

$y = \sin t \qquad dy = \cos t\,dt$

$$I = \int_0^{\frac{\pi}{2}} \left(2\cos t \sin t(-\sin t\,dt)\right) + \cos t\,dt$$

$$= -2\int_0^{\frac{\pi}{2}} \cos t \sin^2 t\,dt + \int_0^{\frac{\pi}{2}} \cos t\,dt$$

$$= -2\int_0^{\pi/2} \sin^2 t \, d\sin t + \int_0^{\pi/2} \cos t \, dt$$

$$= \frac{-2}{3}\left(\sin^3 t\right)_0^{\frac{\pi}{2}} + \left(\sin t\right)_0^{\frac{\pi}{2}}$$

$$= \frac{-2}{3} + 1 = \frac{1}{3}$$

ملحوظة:

$$\frac{d}{dt}\sin t = \cos t$$

$$d\sin t = \cos t \, dt$$